

المباريات الاقليبية في ط د ه
الرياضيات

السنة الثالثة من العلم الثاني الاعداد
المرة الثالثة

تمرين 1

$$\frac{a}{b^2} + \frac{b}{a^2} - \frac{1}{a} - \frac{1}{b} = \frac{a^3 + b^3 - ab^2 - ba^2}{a^2b^2} = \frac{a^2(a-b) - b^2(a-b)}{a^2b^2}$$

$$= \frac{(a-b)(a^2 - b^2)}{a^2b^2} = \frac{(a-b)(a-b)(a+b)}{a^2b^2}$$

$$\frac{a}{b^2} + \frac{b}{a^2} - \frac{1}{a} - \frac{1}{b} = \frac{(a-b)^2(a+b)}{a^2b^2}$$

بما ان $a > 0$ و $b > 0$ ، $a+b > 0$ ، $(a-b)^2 > 0$ ، $a^2b^2 > 0$

$$\frac{a}{b^2} + \frac{b}{a^2} - \frac{1}{a} - \frac{1}{b} > 0 \text{ ، } \frac{(a-b)^2(a+b)}{a^2b^2} > 0 \text{ ، } \text{بما ان}$$

$$\frac{a}{b^2} + \frac{b}{a^2} > \frac{1}{a} + \frac{1}{b} \text{ ، } \text{W.L.G.}$$

تمرين 2

$$\left(a + \frac{1}{a}\right)^2 = 7 \text{ لدينا}$$

$$a + \frac{1}{a} > 0 \text{ ، } \text{بما ان } a + \frac{1}{a} = \sqrt{7} \text{ ، } \text{بما ان}$$

$$\left(a + \frac{1}{a}\right)^2 = 7 \text{ لدينا أيضا ،}$$

$$a^2 + 2 + \frac{1}{a^2} = 7 \text{ يعني}$$

$$a^2 + \frac{1}{a^2} = 5 \text{ يعني}$$

$$a + \frac{1}{a} = \sqrt{5} \text{ ، } \text{بما ان}$$

$$\left(a^2 + \frac{1}{a^2}\right) \left(a + \frac{1}{a}\right) = 5\sqrt{7} \text{ ، } \text{بما ان}$$

$$a^3 + a + \frac{1}{a} + \frac{1}{a^3} = 5\sqrt{7} \text{ ، } \text{بما ان}$$

$$a^3 + \sqrt{7} + \frac{1}{a^3} = 5\sqrt{7} \text{ ، } \text{بما ان}$$

$$\boxed{a^3 + \frac{1}{a^3} = 4\sqrt{7}}$$

(B) $\angle C < \angle A$ و $\angle C < \angle B$ و $\angle C < \angle M$ و $\angle C < \angle E$

(A) $\angle C < \angle A$ و $\angle C < \angle B$ و $\angle C < \angle M$ و $\angle C < \angle E$

$AB < AE + EB$ و $AB < ME + EC$ و $AB < MB + MC$

$EC < ME + MC$ و $ME < MB + MC$

$AB + EC < AE + EB + ME + MC$ و \dots

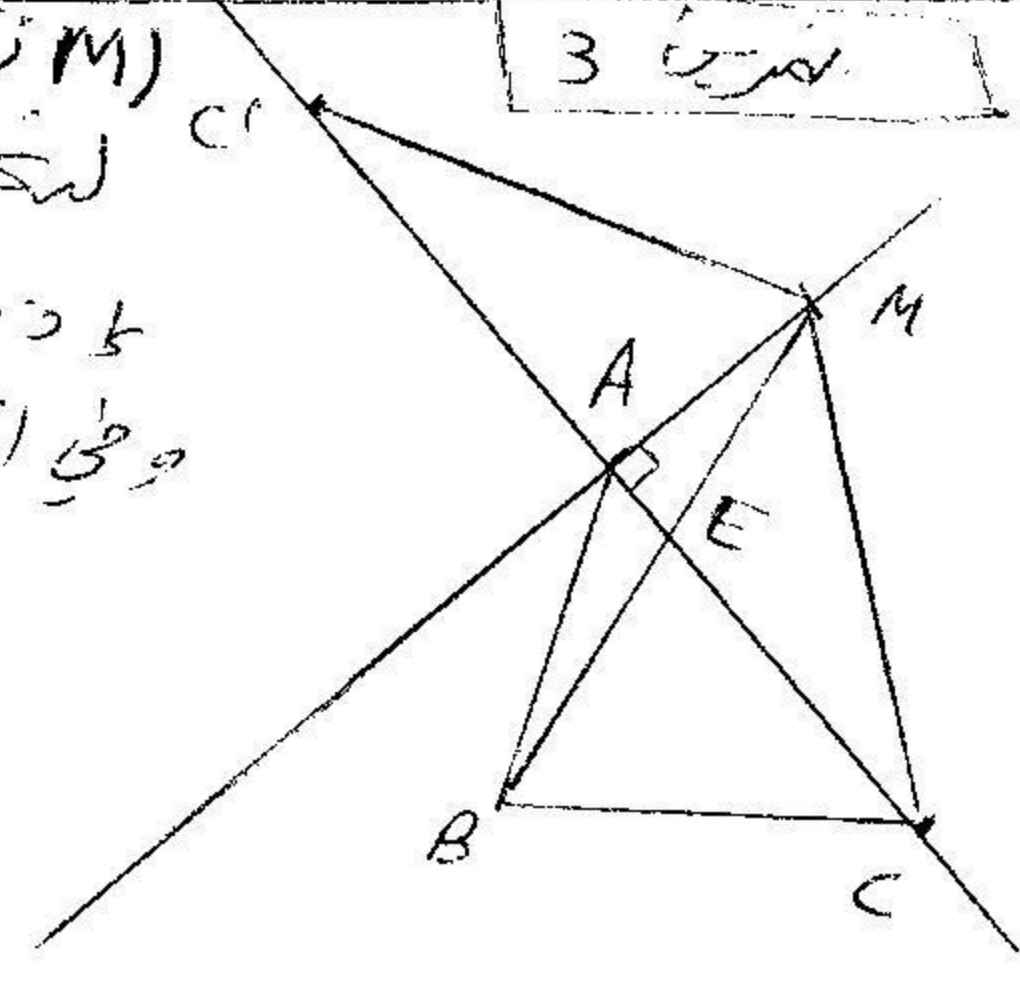
$AB + EC - AE < ME + EB + MC$

$AB + AC < MB + MC$

$AE \in [EC]$ و $EE \in [MB]$ و \dots

$MC = ME$ و $AC = AE$ و \dots

$AB + AC < MB + MC$ و \dots



$$2(\sqrt{n+1} - \sqrt{n}) - \frac{1}{\sqrt{n}} = \frac{2\sqrt{n}(\sqrt{n+1} - \sqrt{n}) - 1}{\sqrt{n}}$$

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$$2(\sqrt{n+1} - \sqrt{n}) - \frac{1}{\sqrt{n}} = \frac{-(n+1 - 2\sqrt{n}(\sqrt{n+1}) + n) - 1}{\sqrt{n}}$$

$$2(\sqrt{n+1} - \sqrt{n}) - \frac{1}{\sqrt{n}} = \frac{-(\sqrt{n+1} - \sqrt{n})^2}{\sqrt{n}}$$

$$\frac{-(\sqrt{n+1} - \sqrt{n})^2}{\sqrt{n}} \leq 0 \Rightarrow \frac{-(\sqrt{n+1} - \sqrt{n})^2}{\sqrt{n}} \leq 0$$

$$2\sqrt{n+1} - \sqrt{n} \leq \frac{1}{\sqrt{n}} \Rightarrow \frac{1}{\sqrt{n}} \leq 2(\sqrt{n+1} - \sqrt{n})$$

$$\boxed{2(\sqrt{n+1} - \sqrt{n}) \leq \frac{1}{\sqrt{n}} \leq 2(\sqrt{n+1} - \sqrt{n})}$$

$$2(\sqrt{2} - \sqrt{1}) \leq 1 \leq 2(\sqrt{2} - \sqrt{1})$$

$$2(\sqrt{3} - \sqrt{2}) \leq \frac{1}{\sqrt{2}} \leq 2(\sqrt{3} - \sqrt{2})$$

$$2(\sqrt{4} - \sqrt{3}) \leq \frac{1}{\sqrt{3}} \leq 2(\sqrt{4} - \sqrt{3})$$

$$2(\sqrt{10000} - \sqrt{9999}) \leq \frac{1}{\sqrt{9999}} \leq 2(\sqrt{10000} - \sqrt{9999})$$

$$2(\sqrt{10001} - \sqrt{10000}) \leq \frac{1}{\sqrt{10000}} \leq 2(\sqrt{10001} - \sqrt{10000})$$

$$2(\sqrt{2} - \sqrt{1} + \sqrt{3} - \sqrt{2} + \dots + \sqrt{10000} - \sqrt{9999} + \sqrt{10001} - \sqrt{10000}) \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{10000}} \leq 2(\sqrt{10001} - \sqrt{10000})$$

$$2(\sqrt{10001} - \sqrt{1}) \leq 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{10000}} \leq 2\sqrt{10001}$$

$$\boxed{2(\sqrt{10001} - 1) \leq 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{10000}} \leq 200}$$